Renormalization Group and Grand Unification with 331 Models

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Abstract

By making a renormalization group analysis we explore the possibility of having a 331 model as the only intermediate gauge group between the standard model and the scale of unification of the three coupling constants. We shall assume that there is no necessarily a group of grand unification at the scale of convergence of the couplings. With this scenario, different 331 models and their corresponding supersymmetric versions are considered, and we find the versions that allow the symmetry breaking described above. Besides, the allowed interval for the 331 symmetry breaking scale, and the behavior of the running coupling constants are obtained. It worths saying that some of the supersymmetric scenarios could be natural frameworks for split supersymmetry. Finally, we look for possible 331 models with a simple group at the grand unification scale, that could fit the symmetry breaking scheme described above.

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1 Introduction

Since the birth of the Standard Model (SM) many attempts have been done to go beyond it, and solve some of the problems of the model such as the charge quantization and the unification of the gauge couplings. In some cases the unification is done by taking a simple group of grand unification, arising the so called Grand Unification Theories (GUT), where the three interactions described by SM are treated as only one [1]-[3], the most common GUT's are $SO(10)$ and E_6. The first condition for these kind of theories is an equal value for the three couplings at certain scale of energy, M_U. This condition cannot be fulfilled by the simplest grand unification schemes with the minimal SM particle content and taking the precision low-energy data. However, the minimal supersymmetric SM can achieve this scenario for the coupling constants [4]-[6]. Other possibilities for unification are the introduction of more degrees of freedom like fermions and scalar fields that lead the three couplings to converge at a high energy scale [7]. Polychromatic extensions of the SM i.e. $SU(N)_C \otimes SU(2)_L \otimes U(1)_{1/N}$ have been considered where the unification of gauge couplings is achieved with $N = 7,\ 5$ and with two, three Higgs doublets, respectively [8]. Alternative proposals of unification of quarks and leptons at TeV scale were considered too [9]. Finally, another interesting alternative consists of enlarging the electroweak sector of the SM gauge group such that the renormalization group equations (RGE) could lead to unification of the three gauge couplings at certain scale M_U, in which there is no necessarily a group of grand unification at the scale of convergence of the couplings. In particular, the model based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge group (hereafter 331 models) is an interesting choice that could address problems like the charge quantization [10]-[11] and the existence of three families based on cancellation of anomalies [12]-[17].

One way to look for new Physics in 331 models is to check for tT production with t denoting the ordinary top quark and T being an exotic quark with charge 2/3 or 4/3 according to the model considered. In LHC the production channels would be $pp \to X^0 \to tT$, when the charge of T is 2/3, and $pp \to X^{++} \to tT$, in the case in which T has an exotic charge 4/3. In the first case, in order to identify the signals, the decay of t is known and identified by the energy spectrum and angular distribution of the final fermions [18]-[20], then this signal is correlated with the one of an exotic particle with the same charge of the top, but with a totally different decay in the final state. The decays of T would be of the form $T \to X^0 t \to \nu E t$, $T \to K^+ b \to \nu E b$, which could de easily identifyable if exotic leptons have already been produced at LHC. In the second case in which T has an exotic

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charge $4/3$, it should be looked for decays of the type $T \to X^{++}t$ where X^{++} is a doubly charged gauge field that should be easily identified if this model is correct. The other possible channel is $T \to K^{+}b \to \nu E b$.

In the scenario of SUSY 331 models, different channels can be searched. A good perspective is $pp \to gg \to g \to \widetilde{g}\widetilde{g}$ where $\widetilde{g} \to \widetilde{t}T, t\widetilde{T}, T\widetilde{T}$ in the case in which T posseses an ordinary charge $2/3$. Another production mechanism in the gaugino sector is could be $pp \to X^{++} \to \chi^{++}\chi^{0}$, $pp \to \chi^{++} \to X^{++}\chi^{0}$, $pp \to Z, Z'^{++}\chi^{--}$ [21]-[25]

In looking for unification of the coupling constant by passing through a 331 model, we shall assume that 1) The 331 gauge group is the only extension of the SM before the unification of the running coupling constants. 2) The hypercharge associated with the 331 gauge group is adequately normalized such that the three gauge couplings unify at certain scale M_U. and 3) There is no necessarily a unified gauge group at the scale of convergence of the couplings M_U. In the absence of a grand unified group, there are no restriction on M_U coming from proton decay[1].

Under our scheme, we have three characteristic energy scales: M_U where the three gauge couplings converge, M_X where the 331 symmetry is broken, and M_Z where the SM breaking occurs. We are going to consider different scenarios for 331 models with one and three families[2], i. e., one family models. We also introduce Supersymmetric versions of the 331 models with different scalar Higgs multiplets. As for the SUSY breaking, we shall consider two different scenarios: when SUSY is broken at the electroweak scale, or when SUSY breaks at the scale of M_X.

In our scheme we have four parameters to take into account, and to look for a possible unification of the coupling constants (UCC). They are the scales M_X, M_U, the value of the coupling contants at the unification convergence point, α_U, and the parameter associated with the normalization of the hypercharge (denoted by a). Since we are interested mostly in possible phenomenological scenarios, the relevant parameter will be the gauge symmetry breaking scale M_X; and the parameters M_U, a can be viewed as functions of this one.

If the unification came from a grand unified symmetry group G, the normalization of the hypercharge Y would be determined by the group structure. However, under our assumptions, this normalization factor is free and the problem could be addressed the opposite way, since the values obtained for a could in turn suggest possible groups of grand unification in which the 331 group is embedded, we shall explore this possibility as well.

In the present work, we study six different versions of 331 models with non-SUSY and SUSY particle content, and find which models could lead to a unification at certain scale M_U with only one symmetry breaking between M_Z and M_U scales. Some of the SUSY versions studied, could provide a quite natural scenario for split supersymmetry. Finally, we also consider the possibility of embedding those 331 versions into a grand unified theory (GUT) in which a gauge group at the unification scale appears.

2 Running Coupling Constants

The evolution for the Running Coupling Constants (RCC) at one-loop order is ruled by the solution of the Renormalization Group Equations (RGE), which can be written in the form [28]:

$$\frac{1}{\alpha_i(\mu_2)} = \frac{1}{\alpha_i(\mu_1)} - \frac{b_i}{2\pi} \ln\left(\frac{\mu_2}{\mu_1}\right), \tag{2.1}$$

where $\alpha_i = g_i^2/4\pi$, and the coefficients b_i are given by [29]:

$$b_i = \frac{2}{3}\sum_f T_{Ri}(f) + \frac{1}{3}\sum_s T_{Ri}(s) - \frac{11}{3}C_{2i}(G). \tag{2.2}$$

The summations run over Weyl fermions and scalars, respectively. The coefficient T_R is the Dynkin index

$$Tr(T^aT^b)_R = T_R\delta^{ab}, \tag{2.3}$$

with the generators in the representation R. The last term is the quadratic Casimir for the adjoint representation

$$f^{acd}f^{bcd} = C_2(G)\delta^{ab}, \tag{2.4}$$

with $C_2(G) = N$ for $SU(N)$. On the other hand, the respective supersymmetric versions are

$$b_i^{SUSY} = \sum_f T_{Ri}(f) + \sum_s T_{Ri}(s) - 3C_{2i}(G). \tag{2.5}$$

where the usual non-supersymmetric degrees of freedom are counted.

[1] Notwithstanding, proton decay could be induced even in the absence of a group of grand unification. It may occurs via six dimensional 331 invariant effective operators, that violates barionic and leptonic numbers [27].

[2] 331 models with three identical fermion multiplets are usually call one family models.

2.1 Matching Conditions

The general expression for the electromagnetic charge operator will be a linear combination of diagonal generators for the gauge group 331:

$$Q = T_3 + Y = T_3 + \frac{2}{\sqrt{3}} b T_8 + X \tag{2.6}$$

with T_i the Gell-Mann matrices normalized as $Tr(T_i T_j) = \frac{1}{2}\delta_{ij}$. The X operator for the abelian group $U(1)_X$ is proportional to the identity matrix 3×3. The hypercharge will be given by

$$Y = \frac{2}{\sqrt{3}} b T_8 + X. \tag{2.7}$$

b is a known parameter that determines the class of 331 models to be considered [30].

The renormalization group analysis compares the couplings for different gauge groups at given energy scales, and models with symmetry breakings need relations for couplings at different energy regions which are called the matching conditions; they are extracted from the way in which the unbroken group is embedded into a bigger broken group. Also, in order to have all couplings in the same ground, all the generators should be normalized in the same way, and well normalized couplings are those which will converge in an unification point.

Calling $\tilde{Y}$ the well-normalized hypercharge operator, it will be proportional to the original one,

$$Y = a\tilde{Y}, \tag{2.8}$$

where a is the normalizing parameter so that the convergence of the running coupling constants at certain scale M_U is guaranteed. In the same way the operator X has a well-normalized $\tilde{X}$, i.e. $X = c\tilde{X}$, and the normalizing parameter for it, is given by Eq. (2.7) requiring the same normalization for $\tilde{Y}$, T_8 which satisfies the following relation

$$a^2 = \frac{4}{3} b^2 + c^2. \tag{2.9}$$

Therefore, the parameter a should be such that

$$a^2 \geq \frac{4}{3} b^2. \tag{2.10}$$

Then, the well-normalized hypercharge operator can be written as a function of the unknown parameter a, as

$$a\tilde{Y} = \frac{2}{\sqrt{3}} b T_8 + \sqrt{\left(a^2 - \frac{4}{3} b^2\right)} \tilde{X}. \tag{2.11}$$

And from this equation, we obtain the following matching condition for the corresponding couplings [33]:

$$a^2 \tilde{\alpha}_Y^{-1} = \frac{4b^2}{3} \alpha_{3L}^{-1} + \left(a^2 - \frac{4}{3} b^2\right) \tilde{\alpha}_X^{-1}. \tag{2.12}$$

where $\tilde{\alpha}_Y$, $\tilde{\alpha}_X$, α_{3L} are related with $U(1)_{\tilde{Y}}$, $U(1)_{\tilde{X}}$ and $SU(3)_L$, respectively.

The following relations must also be satisfied

$$\begin{aligned} \tilde{\alpha}_Y &= a^2 \alpha_Y, \quad \tilde{\alpha}_X = (a^2 - \frac{4b^2}{3}) \alpha_X, \\ \alpha_s &= \alpha_{3C}, \quad \alpha_{2L} = \alpha_{3L}. \end{aligned} \tag{2.13}$$

where α_X, α_Y and α_{2L} are related with $U(1)_X$, $U(1)_Y$ and $SU(2)_L$, respectively. The third relation corresponds to the strong interaction where α_s is associated with the Standard Model and α_{3C} is related with the color part in the 331 model. Finally, the last relation corresponds to the embedding of $SU(2)_L$ into $SU(3)_L$.

2.2 RGE analysis

By replacing the relations described by Eqs. (2.12, 2.13) into Eq. (2.1), we can write the evolution for the RCC from the Z boson-pole M_Z passing through a 331 symmetry breaking scale M_X, up to a certain Scale of unification M_U,

$$
\begin{aligned}
\alpha_U^{-1} &= \frac{1}{a^2 - \frac{4b^2}{3}} \left\{ \alpha_{EM}(M_Z)^{-1} - \frac{4b^2}{3}\alpha_{2L}(M_Z)^{-1} \right. \\
&\qquad \left. - \frac{b_Y - \frac{4b^2}{3}b_{2L}}{2\pi} \ln\left(\frac{M_X}{M_Z}\right) - \frac{b_X}{2\pi}\ln\left(\frac{M_U}{M_X}\right) \right\}, \qquad (2.14) \\
\alpha_U^{-1} &= \alpha_{2L}(M_Z)^{-1} - \frac{b_{2L}}{2\pi}\ln\left(\frac{M_X}{M_Z}\right) - \frac{b_{3L}}{2\pi}\ln\left(\frac{M_U}{M_X}\right), \qquad (2.15) \\
\alpha_U^{-1} &= \alpha_s(M_Z)^{-1} - \frac{b_s}{2\pi}\ln\left(\frac{M_X}{M_Z}\right) - \frac{b_{3C}}{2\pi}\ln\left(\frac{M_U}{M_X}\right). \qquad (2.16)
\end{aligned}
$$

where the coefficients b_s, b_{2L} and b_Y are related with $SU(3)_C, SU(2)_L, U(1)_Y$, respectively and they are calculated at energies in the range $M_Z \leq \mu \leq M_X$. The coefficients b_{3C}, b_{3L}, and b_X, are related with $SU(3)_C, SU(3)_L$, and $U(1)_X$ respectively; they are calculated for energies in the range $M_X \leq \mu \leq M_U$. For our study, we need b_i coefficients for energy scales below (and above) the symmetry breaking scale M_X, which will be given by SM (and 331) degrees of freedom. Then for different models, we have different b_i's for the intervals of energy scales which can change the running of the coupling constants. The input parameters from precision measurements are [31]

$$
\begin{aligned}
\alpha_{EM}^{-1}(M_Z) &= 127.934 \pm 0.027, \\
\sin^2\theta_w(M_Z) &= 0.23113 \pm 0.00015, \\
\alpha_s(M_Z) &= 0.1172 \pm 0.0020, \\
\alpha_{2L}^{-1}(M_Z) &= 29.56938 \pm 0.00068. \qquad (2.17)
\end{aligned}
$$

The M_U scale, where all the well-normalized couplings have the same value, can be calculated from (2.15) and (2.16) as a function of the symmetry breaking scale M_X

$$
M_U = M_X \left(\frac{M_X}{M_Z}\right)^{-\frac{b_s - b_{2L}}{b_{3C} - b_{3L}}} \exp\left\{2\pi \frac{\alpha_s(M_Z)^{-1} - \alpha_{2L}(M_Z)^{-1}}{b_{3C} - b_{3L}}\right\}. \qquad (2.18)
$$

The hierarchy condition $M_X \leq M_U \leq M_{Planck}$, must be satisfied. We shall however impose a stronger condition of $M_U \lesssim 10^{17}$GeV, in order to avoid gravitational effects. Hence, the hierarchy condition becomes

$$
M_X \leq M_U \leq 10^{17} GeV \qquad (2.19)
$$

Such condition can establish an allowed range for the symmetry breaking scale M_X in order to obtain grand unification for a given normalizing parameter a.

With a similar procedure, the expression for a^2 is found, and is given by

$$
\begin{aligned}
a^2 &= \frac{4b^2}{3} + \left\{ \alpha_{EM}(M_Z)^{-1} - \frac{4b^2}{3}\alpha_{2L}(M_Z)^{-1} - \frac{b_Y - \frac{4b^2}{3}b_{2L}}{2\pi}\ln\left(\frac{M_X}{M_Z}\right) \right. \\
&+ \left. b_X \left[\frac{1}{2\pi}\frac{b_s - b_{2L}}{b_{3C} - b_{3L}}\ln\left(\frac{M_X}{M_Z}\right) - \frac{\alpha_s(M_Z)^{-1} - \alpha_{2L}(M_Z)^{-1}}{b_{3C} - b_{3L}} \right] \right\} \\
&\times \left\{ \alpha_{2L}(M_Z)^{-1} - \frac{b_{2L}}{2\pi}\ln\left(\frac{M_X}{M_Z}\right) + b_{3L}\left[\frac{1}{2\pi}\frac{b_s - b_{2L}}{b_{3C} - b_{3L}}\ln\left(\frac{M_X}{M_Z}\right) \right. \right. \\
&- \left. \left. \frac{\alpha_s(M_Z)^{-1} - \alpha_{2L}(M_Z)^{-1}}{b_{3C} - b_{3L}} \right] \right\}^{-1} \\
&\geq \frac{4}{3}b^2. \qquad (2.20)
\end{aligned}
$$

In order to analyze the possibility of having unification at certain scale M_U with 331 as the only intermediate gauge group between the SM and M_U scales, we could distinguish three scenarios of unification pattern

❶ The scenario with $b_{3C} \neq b_{3L}$ and $(b_{3C} - b_{3L}) \neq (b_s - b_{2L})$. We shall call it the first unification pattern (**1UP**). In that case, we obtain an allowed region for the scale M_X. It is carried out by combining the results of Section 2.2. The procedure to get the allowed interval for M_X is described in detail in Section 3.1 for the so called model A.

❷ The scenario with $b_{3C} = b_{3L}$. We call it (**2UP**). In such a case Eq. (2.18) is not valid anymore, and we should go back to Eqs. (2.15, 2.16). For an arbitrary value of M_X the couplings α_{3C} and α_{3L} go parallel each other for energies larger than M_X. Therefore, the only possible way to still obtain unification is by setting $\alpha_{3C} = \alpha_{3L}$ at the scale M_X such that both couplings go together for scales above M_X. Unification with the third coupling could occur at any scale bigger than M_X. By equating Eqs (2.15, 2.16), we find a single value of M_X that makes the couplings α_{3C} and α_{3L} to converge. This convergence occurs at the scale

$$M_X = M_Z \exp\left[\frac{2\pi\left[\alpha_{2L}(M_Z)^{-1} - \alpha_s(M_Z)^{-1}\right]}{(b_{2L} - b_s)}\right] \tag{2.21}$$

It worths emphasizing that this scenario leads to a unique value of M_X and not to an allowed range. Finally, Eq. (2.20) for a^2 must also be recalculated to find

$$\begin{aligned} a^2 &= \frac{F_1(M_X) - \frac{b_X}{2\pi}\ln\left(\frac{M_U}{M_X}\right)}{F_2(M_X) - \frac{b_3}{2\pi}\ln\left(\frac{M_U}{M_X}\right)} + \frac{4b^2}{3} \\ F_1(M_X) &= \alpha_{EM}(M_Z)^{-1} - \frac{4b^2}{3}\alpha_{2L}(M_Z)^{-1} - \frac{b_Y - \frac{4b^2}{3}b_{2L}}{2\pi}\left\{\frac{2\pi\left[\alpha_{2L}(M_Z)^{-1} - \alpha_s(M_Z)^{-1}\right]}{(b_{2L} - b_s)}\right\} \\ F_2(M_X) &= \alpha_{2L}(M_Z)^{-1} - \frac{b_{2L}}{2\pi}\left\{\frac{2\pi\left[\alpha_{2L}(M_Z)^{-1} - \alpha_s(M_Z)^{-1}\right]}{(b_{2L} - b_s)}\right\} \end{aligned} \tag{2.22}$$

❸ The **3UP** with $(b_{3C} - b_{3L}) = (b_s - b_{2L}) \neq 0$; according to Eq. (2.18), the unification scale M_U becomes independent of M_X.

The case $(b_{3C} - b_{3L}) = (b_s - b_{2L}) = 0$, does not lead to unification as can be seen by trying to equate Eqs. (2.15) and (2.16). Since the first scenario is the most commom one, we shall only indicate when the other two scenarios appear. We will study non-SUSY and SUSY versions of the 331 models. In the case of SUSY models we shall consider two scenarios for the SUSY breaking pattern

1. The SUSY Breaking Scenario at the $Z-$pole (**ZSBS**), in which the SUSY breaking scale is taken as $\Lambda_{SUSY} \sim M_Z$. Although this is not a very realistic scenario, numerical results do not change significantly with respect to the more realistic scenario with Λ_{SUSY} lying at some few TeV's.

2. The SUSY Breaking Scenario at the M_X scale (**XSBS**), with $\Lambda_{SUSY} \sim M_X$ i.e. SUSY breaking at the 331 breaking scale.

3 331 Models

Analogously to the SM, fermions will transform as singlets or in the fundamental representation of $SU(3)_L$, and gauge fields in the adjoint representation. Assignment of $U(1)_X$ quantum numbers should be done ensuring a model free of anomalies. There are models with three different families necessary to cancel out the anomalies, and there are models with only one family and the other two are a copy of the first. We will take into account six different versions of the 331 model for the analysis of the unification scheme.

The minimal spectrum necessary for symmetry breaking and generation of masses is given by [32]

$$\begin{aligned} &\phi_1 \sim (1, 3^*, -1/3), \\ &\phi_2 \sim (1, 3^*, -1/3), \\ &\phi_3 \sim (1, 3^*, 2/3). \end{aligned} \tag{3.1}$$

Where the quantum numbers are associated with $SU(3)_C$, $SU(3)_L$, and $U(1)_X$ respectively. The first multiplet acquires a vacuum expectation value (VEV) at M_X scale, breaking the symmetry as $331 \to 321$; the other two will

be decomposed as singlets and doublets of $SU(2)_L$, which will break the 321 symmetry. The spectrum transforms like

$$\phi_2 \quad \rightarrow \quad \begin{cases} \phi_{2SM} & \sim (1, 2^*, -1/2), \\ \phi_2^0 & \sim (1, 1, 0). \end{cases} \tag{3.2}$$

$$\phi_3 \quad \rightarrow \quad \begin{cases} \phi_{3SM} & \sim (1, 2^*, 1/2), \\ \phi_2^+ & \sim (1, 1, 1). \end{cases} \tag{3.3}$$

This is applied for models with $b = 1/2$, with tiny changes that do not affect our analysis [43, 44]. For models with $b = 3/2$ this spectrum should be varied in order to get a phenomenological mass spectrum. For this case we have the following transformation for the triplets [26, 34, 35, 36]:

$$\begin{aligned} \eta &\sim (1, 3^*, 0), \\ \rho &\sim (1, 3^*, 1), \\ \chi &\sim (1, 3^*, -1). \end{aligned} \tag{3.4}$$

the 331 symmetry is broken when χ acquires a VEV in the third component. The remaining scalars will transform in the 321 symmetry as

$$\eta \quad \rightarrow \quad \begin{cases} \eta_{SM} & \sim (1, 2^*, -1/2), \\ \eta_2^+ & \sim (1, 1, 1). \end{cases} \tag{3.5}$$

$$\rho \quad \rightarrow \quad \begin{cases} \rho_{SM} & \sim (1, 2^*, 1/2), \\ \rho^{++} & \sim (1, 1, 2). \end{cases} \tag{3.6}$$

The Model that we will call Model E needs also another scalar boson transforming under $\underline{6}$ representation of $SU(3)_L$. Such an structure is required in order to give masses to the neutrinos [26, 34, 35],

$$S \sim (1, 6, 0) \tag{3.7}$$

which has the following representation in the 321 symmetry:

$$S \rightarrow \begin{cases} \phi_{3SM} & \sim (1, 3, 1), \\ \phi_{2SM} & \sim (1, 2, 1/2), \\ \phi^{--} & \sim (1, 1, -2). \end{cases} \tag{3.8}$$

3.1 Model A

The simplest anomaly-free structure for this gauge group is generated by taking $b = 1/2$; and the following spectrum is obtained [37]

$$\begin{aligned} \psi_{1L} &= (e^-, \nu_e, N_1^0)_L^T \sim (1, 3^*, -1/3), \\ \psi_{2L} &= (E^-, N_2^0, N_3^0)_L^T \sim (1, 3^*, -1/3), \\ \psi_{3L} &= (N_4^0, E^+, e^+)_L^T \sim (1, 3^*, 2/3), \\ Q_L &= (u, d, D)_L^T \sim (3, 3, 0), \\ u_L^c &\sim (3^*, 1, -2/3), \quad d_L^c \sim (3^*, 1, 1/3), \quad D_L^c \sim (3^*, 1, 1/3). \end{aligned} \tag{3.9}$$

The other two families are copies of the first one, and each family is free of anomalies. This particular 331 model could be embbeded in the E_6 gauge theory, but we shall assume that not necessarily such embedding occurs.

From the general expression for coefficients b_i Eq. (2.2), and using the quantum numbers assigned for each representation in this model, they can be expressed at energies below M_X in the following form

$$\begin{aligned} b_Y &= \frac{20}{9} N_g + \frac{1}{6} N_H + \frac{1}{3} \sum_{sing-s} Y^2(s), \\ b_{2L} &= \frac{4}{3} N_g + \frac{1}{6} N_H - \frac{22}{3}, \\ b_s &= \frac{4}{3} N_g - 11. \end{aligned} \tag{3.10}$$

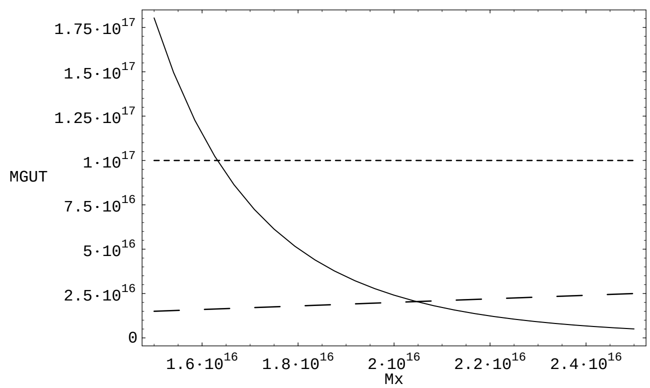

Figure 1: Allowed interval for M_X in model A. The solid line represents M_U as a function of M_X. The horizontal short dashed line is the graphics for $M_U = 10^{17}$GeV. The long dash line represents the function $M_U = M_X$.

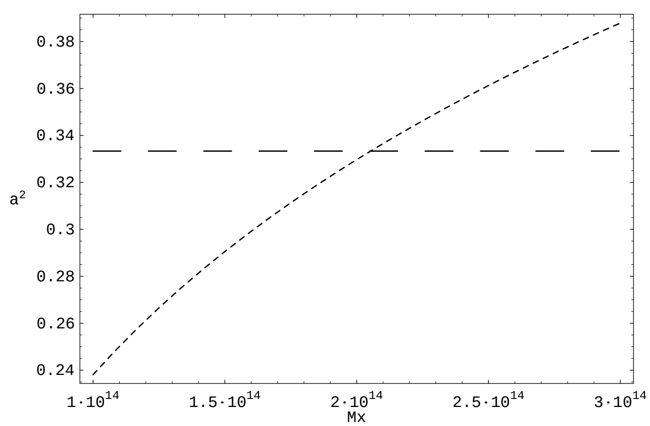

Figure 2: Normalization parameter a^2 as function of the symmetry breaking scale M_X for model A. The intersection of this function with the constant function $\frac{4}{3}b^2$ gives a lower limit for M_X owing to the condition described by Eq. (2.10).

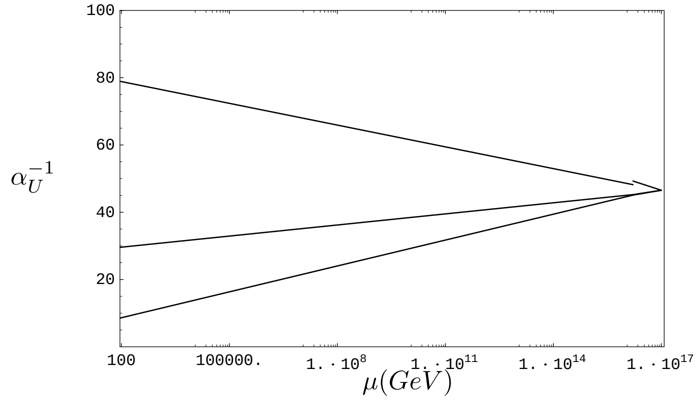

Figure 3: Running Coupling Constants for model A with $M_X = 1.63 \times 10^{16} GeV$. The unification scale appears at $M_U \simeq 10^{17}$GeV

where N_g is the number of families, N_H the number of scalar doublets in $SU(2)_L$, and the sum runs over scalar singlets.

In the same way, the b_i's for energies above the M_X are given by

$$\begin{aligned} b_X &= \frac{8}{3}N_g + \frac{2}{3}, \\ b_{3L} &= 2N_g + \frac{1}{2} - 11, \\ b_{3C} &= 2N_g - 11. \end{aligned} \tag{3.11}$$

Therefore, from the spectrum showed in Eq. (3.9), we obtain

$$(b_Y, b_{2L}, b_s) = \left(\frac{22}{3}, -3, -7\right) \tag{3.12}$$

for energies below M_X, and

$$(b_X, b_{3L}, b_{3C}) = \left(\frac{26}{3}, -\frac{9}{2}, -5\right) \tag{3.13}$$

for energies above M_X.

First of all, by using the hierarchy condition Eq. (2.19), an allowed region for M_X can be found. In Fig. 1 we plot M_U as a function of M_X, Eq. (2.18). We also plot the line $M_U = M_X$, and the line $M_U = 10^{17}$GeV, the intersection of these lines with the first plot, provides the allowed region given by the constraints $M_X \leq M_U \leq 10^{17}$GeV according to Eq. (2.19). The intersection between the curve $M_U(M_X)$ from Eq. (2.18) with $M_U = 10^{17}$ GeV gives the lower bound for M_X, and the intersection with $M_U = M_X$ gives the upper limit for M_X. Then from Fig. 1 and the hierarchy condition, it is obtained that

$$1.63 \times 10^{16} GeV \leq M_X \leq 2.05 \times 10^{16} GeV. \tag{3.14}$$

In Figure 2 we plot a^2 as a function of M_X from Eq. (2.20). For this model $b = 1/2$ and from Eq. (2.10) the restriction $a^2 \geq 1/3$ is obtained. The horizontal line $a^2 = 1/3$ is also showed and the allowed values are above of this line. In this case the constraint for M_X is given by $M_X \leq 2 \times 10^{14}$ GeV which is weaker than the previous restriction.

From Eq. (2.18) and the allowed interval for M_X of Eq. (3.14) we get the region permitted for M_U i.e. 2×10^{16} GeV $\leq M_U \leq 10^{17}$GeV.

Now, for $M_X = 2.05 \times 10^{16}$GeV (the highest allowed value) we obtain from Eq. (2.18) that $M_U \approx M_X$ which is not a natural hierarchy. Instead, for $M_X = 1.63 \times 10^{16}$GeV (the lowest allowed value) the unification occurs at a more natural scale of 10^{17}GeV. Then, we shall take the latter to plot the evolution of the running coupling constants.

The evolution of the RCC described by Eqs. (2.1), requires the results for the matching conditions Eqs. (2.12, 2.13). Moreover, the following input parameters are necessary: ① The b_i coefficients Eqs. (3.12, 3.13), ② The value of the 331 breaking scale M_X, and ③ the values of the couplings at Z-pole Eqs. (2.17). Assuming the lowest allowed value for the 331 breaking scale ($M_X = 1.63 \times 10^{16}$GeV) we plot the evolution of the running coupling constants in Fig. 3. The unification scale appears to be $M_U = 10^{17}$GeV, in agreement with our previous results.

3.1.1 SUSY Extension

One possible SUSY extension in which the superpartners cancel the anomalies of their partners, arises by introducing new scalars which transform in the conjugate representations of the original ones. Under these considerations, and using Eqs. (2.5) we obtain the supersymmetric b_i coefficients

$$(b_Y, b_{2L}, b_s) = (14, 2, -3), \tag{3.15}$$
$$(b_X, b_{3L}, b_{3C}) = (16, 3, 0). \tag{3.16}$$

We take first the **ZSBS** in which SUSY breaking is assumed to occur at EW scale [3]. Since in that case SUSY is not broken by going from EW to the unification scale, we should use the supersymmetric b_i coefficients, at scales

[3] Our results are also valid for SUSY breaking at scales of the order of 1 TeV.

below and above the 331 breaking scale. Taking it into account, and using the hierarchy condition (2.19), as it was explained in the previous section, we get an allowed range for M_X, of

$$1.27 \times 10^8 \text{GeV} \leq M_X \leq 2.76 \times 10^{13} \text{GeV}. \tag{3.17}$$

The bound coming from the a parameter Eq. (2.10), yields $6.43 \times 10^6 GeV \leq M_X$, which gives no further restriction. This allowed region for M_X leads to an allowed interval for M_U obtained from Eq. (2.18), getting[4]

$$2.76 \times 10^{13} \text{GeV} \leq M_U \leq 10^{17} \text{GeV}$$

On the other hand, we shall consider the **XSBS** with SUSY breaking at the M_X scale. In that case, we should use the non-SUSY b_i coefficients for scales below M_X, and the SUSY b_i coefficients for energies above M_X. From this we obtain the following allowed interval for M_X

$$1.76 \times 10^{14} \text{GeV} \leq M_X \leq 2.05 \times 10^{16} \text{GeV}, \tag{3.18}$$

for which the restriction $a^2 > 1/3$ is also satisfied. This allowed region provides a unification scale between $2.05 \times 10^{16} GeV$ and $10^{17} GeV$ which correspond to the values calculated from Eq. (2.18) for the lower and upper values for M_X, respectively. Therefore, this is a possible scenario of unification with 331 as the unique intermediate gauge group, for both schemes of SUSY breaking.

It worths noting that SUSY scenarios leads to much lower breaking scales for the 331 model, especially for low energy SUSY breaking.

In another possible supersymmetric extension of model A, the scalar fields are not introduced explicitly. Instead, they appear as superpartners of the leptons, since they have the same representations and quantum numbers [45]. The supersymmetric b_i coefficients read

$$\begin{aligned} (b_Y, b_{2L}, b_s) &= (10, 0, -3)\,, & (3.19)\\ (b_X, b_{3L}, b_{3C}) &= (12, 0, 0)\,. & (3.20) \end{aligned}$$

For both schemes of SUSY breaking, the model correspond to the **2UP** explained in Sec. (2.2), so that M_X acquires a single value. The 331 scale appears at $M_X \simeq 10^{21}$GeV. It discards this SUSY version of model A, to get **UCC** under our scheme.

3.2 Model B

Again the parameter b takes the value $1/2$, and in this case the 331 gauge theory is a family-symmetric model. For each family the spectrum is [38]:

$$\begin{aligned} \psi_{1L} &= (e^-, \nu_e, E_1^-)_L^T \sim (1, 3^*, -2/3), \\ \psi_{2L} &= (N1^0, E_2^+, \nu_e^c)_L^T \sim (1, 3^*, 1/3), \\ \psi_{3L} &= (E_2^-, N_2^0, E_3^-)_L^T \sim (1, 3^*, 2/3), \\ e_L^+ &\sim (1,1,1), \quad E_{1L}^+ \sim (1,1,1), \quad E_{3L}^+ \sim (1,1,1), \\ Q_L &= (u, d, U)_L^T \sim (3, 3, 1/3), \\ u_L^c &\sim (3^*, 1, -2/3), \quad d_L^c \sim (3^*, 1, 1/3), \quad U_L^c \sim (3^*, 1, -2/3). \end{aligned} \tag{3.21}$$

In this case, the 331 gauge group could be embbeded into the $SU(6) \otimes U(1)_X$ gauge theory, but we suppose it is not mandatory. This model has the same spectrum of model A at low energies. Therefore, it has the same values for the b_i coefficients at low energies given by Eq. (3.12)

$$(b_Y, b_{2L}, b_s) = \left(\frac{22}{3}, -3, -7\right). \tag{3.22}$$

For energies larger than M_X, we have that b_{3L} and b_{3C} are the same as in model A, since their values are ruled by the number of triplets, which coincide for both models. For b_X, there is a difference coming from the additional singlets in the lepton sector. In this case, we obtain

$$b_X = \frac{20}{3} N_g + \frac{2}{3} = \frac{62}{3}.$$

[4] When a group of grand unification is present, protection from proton decay leads to a stronger bound for M_X by demanding $M_U \gtrsim 2 \times 10^{16}$GeV, in that case we find $1.27 \times 10^8 \text{GeV} \leq M_X \leq 1.4 \times 10^9$GeV.

Hence,

$$(b_X, b_{3L}, b_{3C}) = \left(\frac{62}{3}, -\frac{9}{2}, -5\right). \tag{3.23}$$

Following the same procedure as for the Model A, we find the allowed range for M_X

$$1.63 \times 10^{16}\text{GeV} \leq M_X \leq 2.05 \times 10^{16}\text{GeV}. \tag{3.24}$$

In this interval, $a^2 > 1/3$ yields $M_X \geq 3.42 \times 10^{15}$ GeV, and M_U lies in the interval $\left[2.05 \times 10^{16}, 10^{17}\right]$GeV.

3.2.1 SUSY Extension

As a first attempt, we take the model in Ref. [39], where scalars are considered as sleptons and two new superfields are introduced in order to give mass to the supersymmetric particles and cancel out the anomalies. They transform as

$$\hat{\psi}_{4L} \sim (1, 3, 1/3), \quad \hat{\psi}_{5L} \sim (1, 3^*, -1/3). \tag{3.25}$$

The spectrum is the same at low energies as SUSY model A, Eq. (3.19). The supersymmetric b_i coefficients read

$$(b_Y, b_{2L}, b_s) = (10, 0, -3)\,, \tag{3.26a}$$
$$(b_X, b_{3L}, b_{3C}) = (24, 3, 0)\,. \tag{3.26b}$$

For the **ZSBS** ($\Lambda_{SUSY} \approx M_Z$), we are in the **3UP** explained in Sec. (2.2) so that M_U acquires a single value

$$M_U = 1.24 \times 10^{21} GeV. \tag{3.27}$$

which discards this SUSY version of model B with $\Lambda_{SUSY} \approx M_Z$, to achieve unification with our breaking scheme.

In the **XSBS** ($\Lambda_{SUSY} \approx M_X$), the allowed region for the model is

$$1.76 \times 10^{14}\text{GeV} \leq M_X \leq 2.05 \times 10^{16}\text{GeV}. \tag{3.28}$$

the bound from a^2 is $M_X \geq 1.44{\times}10^{11}$GeV giving no further restriction. M_U belongs to the interval $\left[2.05 \times 10^{16},\ 10^{17}\right]$GeV. Hence, this SUSY version of model B with $\Lambda_{SUSY} \approx M_X$ permits **UCC** under our assumptions.

3.3 Model C

In this model, the parameter b is also equal to 1/2, but the cancellation of anomalies is obtained with a number of families multiple of three [40]. Its fermionic spectrum is

$$\begin{aligned}
\psi_L^\alpha &= (\nu_\alpha, \alpha^-, E_\alpha^-)_L^T \sim (1, 3, -2/3), \\
\alpha_L^+ &\sim (1, 1, 1), \quad E_{\alpha L}^+ \sim (1, 1, 1), \\
Q_L^i &= (d^i, u^i, U^i)_L^T \sim (3, 3^*, 1/3), \\
u_L^{ic} &\sim (3^*, 1, -2/3), \quad d_L^{ic} \sim (3^*, 1, 1/3), \quad U_L^{ic} \sim (3^*, 1, -2/3), \\
Q_L^3 &= (u_3, d_3, D)_L^T \sim (3, 3, 0), \\
u_{3L}^c &\sim (3^*, 1, -2/3), \quad d_{3L}^c \sim (3^*, 1, 1/3), \quad D_L^c \sim (3^*, 1, /3).
\end{aligned} \tag{3.29}$$

where $\alpha = 1, 2, 3$ label the three lepton families; while $i = 1, 2$ refer to only two of the quark families because the third one transforms in a different way. The spectrum is the same of the Model A at low energies, so that the b_i coefficients are given by Eq. (3.12). For energies above M_X there are new contributions and the b_i's yield

$$\begin{aligned}
(b_Y, b_{2L}, b_s) &= \left(\frac{22}{3}, -3, -7\right) \\
(b_X, b_{3L}, b_{3C}) &= \left(\frac{42}{3}, -\frac{13}{2}, -5\right).
\end{aligned} \tag{3.30}$$

Following the same procedure explained before, we find that the allowed region for M_X is

$$2.05 \times 10^{16} GeV \leq M_X \leq 3.15 \times 10^{16} GeV. \tag{3.31}$$

In this case $a^2 \geq 1/3$, leads to $M_X \leq 9.57 \times 10^{18}$GeV and provides an upper (and not a lower) limit because a^2 is a decreasing function of M_X. Of course, this constraint gives no further restrictions. Taking values for M_X from the interval in Eq. (3.31), the permitted interval for M_U is $\left[2.05 \times 10^{16}\text{GeV}, 10^{17}\text{GeV}\right]$.

3.3.1 SUSY Extension

The extension is done adding new scalars transforming in the conjugate representation in order to cancel the anomalies. With this spectrum we obtain

$$\begin{aligned}(b_Y, b_{2L}, b_s) &= (14, 2, -3)\\ (b_X, b_{3L}, b_{3C}) &= (24, 0, 0)\,.\end{aligned} \tag{3.32}$$

At low energy scales, the coefficients coincide with the analogous in model A, Eq. (3.15). For both schemes of SUSY breaking we lie in the **2UP** getting a unique M_X from Eq. (2.21). For the **ZSBS**, M_X becomes

$$M_X = 2.76 \times 10^{13} GeV. \tag{3.33}$$

If we fit $M_U = 2.76 \times 10^{13}\left(= 1 \times 10^{17}\right)$ GeV, we need $a^2 = 3.26\,(= 1.78)$ to obtain unification of the three couplings according to Eq. (2.22). In general, for M_U lying in the interval $\left[2.76 \times 10^{13}\text{GeV},\ 10^{17}\text{ GeV}\right]$, we find that the a^2 parameter lies in the interval $[1.78,\ 3.26]$; and the condition $a^2 \geq (4/3)\,b^2$ is satisfied. Therefore, this model gives a consistent unification with 331 as the only intermediate gauge group.

For the **XSBS**, $M_X = 2.05 \times 10^{16}$ GeV. From $M_U \in \left[2.05 \times 10^{16},\ 1 \times 10^{17}\right]$ GeV, we get $a^2 \in [1.84,\ 1.97]$ and $a^2 \geq (4/3)\,b^2$ is fulfilled in all cases and the model is viable as well.

3.4 Model D

This model differs from model C just in its spectrum [41, 42], which is given by

$$\begin{aligned}\psi_L^\alpha &= (\alpha^-, \nu_\alpha, N_\alpha^0)_L^T \sim (1, 3, -1/3),\\ \alpha_L^+ &\sim (1, 1, -1),\\ Q_L^i &= (d^i, -u^i, D^i)_L^T \sim (3, 3^*, 0),\\ u_L^{ic} &\sim (3, 1, +2/3), \quad d_L^{ic} \sim (3, 1, -1/3), \quad D_L^{ic} \sim (3, 1, -1/3),\\ Q_L^3 &= (d_3, u_3, U)_L^T \sim (3, 3, 1/3),\\ u_{3L}^c &\sim (3, 1, 2/3), \quad d_{3L}^c \sim (3, 1, -1/3), \quad U_L^c \sim (3, 1, 2/3).\end{aligned} \tag{3.34}$$

with $\alpha = 1, 2, 3$, running over the three families while $i = 1, 2$ runs over two families. The model differs from the others above in its spectrum at high energies. Therefore for the 331 gauge group the b_i functions coincide with the values in the Eq. (3.12),

$$(b_Y, b_{2L}, b_s) = \left(\frac{22}{3}, -3, -7\right). \tag{3.35}$$

For energies larger than the symmetry breaking scale M_X we have

$$(b_X, b_{3L}, b_{3C}) = \left(\frac{26}{3}, -\frac{13}{2}, -5\right). \tag{3.36}$$

These values give the following allowed region

$$2.05 \times 10^{16} GeV \leq M_X \leq 3.15 \times 10^{16} GeV, \tag{3.37}$$

and $a^2 > 1/3$ predicts $M_X \leq 1.54 \times 10^{20}$GeV, yielding no further restriction. The scale of grand unification lies in the interval $2.05 \times 10^{16}\text{GeV} \leq M_U \leq 10^{17}$GeV.

3.4.1 SUSY Extension

Treating the model in the same way as model C, the b_i coefficients at low energies are given in Eq. (3.15), and for high energy scales they are given by

$$(b_X, b_{3L}, b_{3C}) = (16, 0, 0)\,. \tag{3.38}$$

Both schemes of SUSY breaking are in the **2UP**. For the **ZSBS** we find $M_X = 2.76 \times 10^{13} GeV$. Taking $M_U \in \left[2.76 \times 10^{13},\ 1 \times 10^{17}\right]$ GeV we obtain $a^2 \in [2.27,\ 3.26]$, and the basic restriction on a^2 is always accomplished. With **XSBS**, it is found that $M_X = 2.05 \times 10^{16}$, and we get $M_U \in \left[2 \times 10^{16},\ 1 \times 10^{17}\right]$ GeV for $a^2 = [1.88,\ 1.97]$, respectively.

Thus the SUSY version provides a possible scenario of unification for both schemes of SUSY breaking.

3.5 Model E

In this model $b = 3/2$ and the cancellation of anomalies is obtained by the interplay of three families [26, 34, 35]. Its fermionic spectrum is

$$\begin{aligned}
\psi_L^\alpha &= (e^\alpha, \nu^\alpha, e^{c\alpha})_l^T \sim (1, 3^*, 0), \\
q_L^i &= (u^i, d^i, j^i)_L^T \sim (3, 3, -1/3), \\
q_L^1 &= (d^1, u^1, s)_L^T \sim (3, 3^*, 2/3), \\
u_L^{c\alpha} &\sim (3, 1, -2/3), \quad d_L^{c\alpha} \sim (3, 1, 1/3), \quad s_L^c \sim (3, 1, -5/3), \\
j_L^{ci} &\sim (3, 1, 4/3).
\end{aligned} \tag{3.39}$$

where $\alpha = 1, 2, 3$ labels the families, and $i = 2, 3$ are related with two of them.

Taking into account the whole possible scalar spectrum at low energy scales, including the scalars in the 6-dimensional representation, we can write the b_i coefficients as

$$\begin{aligned}
b_Y &= \frac{20}{9} N_g + \frac{1}{6} N_H + \frac{1}{3} 3Y^2(triplet) + \frac{1}{3} \sum_{singl-s} Y^2(s), \\
b_{2L} &= \frac{4}{3} N_g + \frac{1}{6} N_H + \frac{1}{3} T_R(triplet) - \frac{22}{3}, \\
b_s &= \frac{4}{3} N_g - 11.
\end{aligned} \tag{3.40}$$

where $Y(triplet)$ and $T_R(triplet)$ mean the quantum numbers of the $SU(2)_L$ scalar triplet in the 6-dimensional representation. Therefore the values of the b_i coefficients read

$$(b_Y, b_{2L}, b_s) = \left(\frac{23}{2}, -\frac{13}{6}, -7\right), \tag{3.41}$$

$$(b_X, b_{3L}, b_{3C}) = \left(22, -\frac{17}{3}, -5\right). \tag{3.42}$$

With these b_i coefficients we only find a lower limit from the hierarchy condition,

$$6.87 \times 10^{13} GeV \leq M_X \tag{3.43}$$

An upper limit is obtained from Eq. (2.10) with $3 \leq a^2$, which is satisfied for $M_X \leq 6.03 \times 10^{12} GeV$, but both constraints are inconsistent and rule out the model under our assumptions.

3.5.1 SUSY Extension

The supersymmetric model is constructed adding scalar fields in the complex representation in order to avoid chiral anomalies from their superpartners. With this spectrum we obtain the following values for the b_i coefficients

$$\begin{aligned}
(b_Y, b_{2L}, b_s) &= (28, 6, -3), \\
(b_X, b_{3L}, b_{3C}) &= (42, 5, 0).
\end{aligned} \tag{3.44}$$

We also consider an extended spectrum for the scalar sector. With these values we get only an upper limit for M_X in the **ZSBS**

$$M_X \leq 2.18 \times 10^8 \text{GeV}. \tag{3.45}$$

But the restriction in the normalization parameter a in Eq. (2.10), requires $M_X \geq 8.1 \times 10^8$. This situation discards the model with only one intermediate breaking.

For the **XSBS**, the hierarchy condition gives only a lower limit 6.87×10^{13}GeV$\leq M_X$. But $a^2 \geq 3$ gives $M_X \geq 1.35 \times 10^{22} GeV$, discarding the model.

3.6 Model F

It is also possible to obtain cancellation of anomalies by a tiny variation of the fermionic spectrum from the model D of Pleitez-Tonasse [36]. This new spectrum will be given by

$$\begin{aligned}\psi_L^{\alpha} &= (\nu^{\alpha}, e^{\alpha -}, E^{\alpha +})_l^T \sim (1,3,0), \\ e_L^{\alpha +} &\sim (1,1,-1), \quad E_L^{\alpha +} \sim (1,1,1).\end{aligned} \tag{3.46}$$

where an exotic lepton has been introduced in the third entry of the triplet instead of the electron right-handed part, such that there are also leptonic singlets of $SU(3)_L$.

The b_i coefficients at low energy scales differ from those for the model E in the contribution due to the sextuplet. Therefore the values are

$$\begin{aligned}(b_Y, b_{2L}, b_s) &= (26/3, -3, -7), \\ (b_X, b_{3L}, b_{3C}) &= \left(26, -\frac{13}{2}, -5\right).\end{aligned} \tag{3.47}$$

With these coefficients, the allowed region for M_X is given by

$$2.05 \times 10^{16} GeV \leq M_X \leq 3.15 \times 10^{16} GeV. \tag{3.48}$$

The restriction from Eq. (2.10) concerning the parameter a^2, gives an upper bound of $M_X \leq 4.1 \times 10^{17} GeV$, i.e. no additional restriction.

3.6.1 SUSY Extension

The extension is done analogously to the model E, obtaining the following values for the b_i coefficients

$$\begin{aligned}(b_Y, b_{2L}, b_s) &= (22, 2, -3), \\ (b_X, b_{3L}, b_{3C}) &= (48, 0, 0).\end{aligned} \tag{3.49}$$

The SUSY version is in the **2UP** for both breaking schemes. In the **ZSBS**, M_X reads

$$M_X = 2.76 \times 10^{13} GeV. \tag{3.50}$$

In this case, the condition $a^2 \geq 1/3$ imposes a strong bound on the M_U scale. From

$$2.76 \times 10^{13} \mathrm{GeV} \leq M_U \leq 1.13 \times 10^{15} \mathrm{GeV}$$

we get $a^2 = [1/3,\ 1.67]$. Larger values of M_U leads to non allowed values of a^2 (less than 1/3).

In the **XSBS** we have $M_X = 2.05 \times 10^{16} GeV$. Using $M_U \in \left[2 \times 10^{16},\ 10^{17}\right]$ GeV we get $a^2 \in [1.55,\ 1.82]$. They satisfy the condition $a^2 \geq 1/3$. Hence, SUSY versions give a possible scenario of unification in our scheme.

4 Some possible scenarios with groups of grand unification

One of our main assumptions was that there is no necessarily a group of grand unification at the scale in which the coupling constants converge. In this way, the a^2 parameter becomes free, and indeed we could reverse the problem in a certain way, since we have in many cases an allowed region for a^2 and we could ask what groups of grand unification (if any) could lead to values of a^2 admitted by our scheme. Let us elaborate about this possibility

After working some 331 models with one or three families, we can see that in some of them is possible to find a scale M_X and a normalization factor a^2 that gives unification of the coupling constants. In the absence of a group of grand unification (GUT) a^2 is not fixed by the group structure. Notwithstanding, UCC imposes some restrictions on a^2, and by using these values we can look for simple groups containing a 331 group and fixing an a^2 belonging to the allowed interval mentioned above. Some good examples of GUT are E_6, and $SU(7)$.

The group E_6 can be broken into $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ or $SU(6)_L \otimes SU(2)$. Models A and B shown in this paper could be embedded in this scheme. On the other hand, the 331 models of three families can be embedded into $SU(7)$, thus, we shall study the latter scenario in more detail.

There are different versions of the $SU(7)$ GUT that lead to 331 models according to the irreducible representations (irrep) that cancel anomalies, and the definition of the electric charge or equivalently the linear combination of diagonal generators that define the hypercharge (i.e. the normalizing factor a^2). The combination of irreducible representations free of anomalies, permits to accomodate those 331 models with three families by the branching rules. The additional singlets or exotic fields acquire masses over the M_X scale, and do not affect the RGE. There are different combinations of anomaly free irreps. Taking into account the irreps $\Psi^{\alpha}_{(1)}[7]$, $\Psi^{\alpha\beta}_{(3)}[21]$ and $\Psi^{\alpha\beta\gamma}_{(2)}[35]$, where the subindex means the anomaly coefficient, the bracket coefficient means the dimension and the labels $\alpha,\beta,\gamma = 1,\cdots,7$. Models can be classified if no irrep appears more than once i.e. a form like $\Psi_{(-1)\alpha} \oplus \Psi^{\alpha\beta}_{(3)} \oplus \Psi_{(-2)\alpha\beta\gamma}$; but in general the same irrep can be repeated such as $5 \times \Psi_{(-1)\alpha} \oplus \Psi^{\alpha\beta}_{(3)} \oplus \Psi^{\alpha\beta\gamma}_{(2)}$. On the other hand, the branching rules according to $(SU(3)_c, SU(2)_L)$ are given by

$$\begin{aligned}
\Psi^{\alpha} \oplus \Psi^{\alpha\beta} \oplus \Psi^{\alpha\beta\gamma} &= [(3^*,1)+(1,2)+(1,1)+(1,1)]_7 \oplus [(3^*,1)+(3,2) \\
&+ (3,1)+(3,1)+(1,1)+(1,2)+(1,2)+(1,1)]_{21} \\
&\oplus [(1,1)+(3,2)+(3,1)+(3,1)+(3^*,1)+(3^*,2) \\
&+ (3^*,2)+(3^*,1)+(1,1)+(1,1)+(1,2)]_{35}
\end{aligned} \tag{4.1}$$

The electromagnetic charge of the particles can be chosen by defining the hypercharge which is a linear combination of the $U(1)$ factors of the $SU(7)$, and imposing the condition that the singlet of color have electromagnetic charges $q = \pm 1/3, \pm 2/3, \pm 4/3, \pm 5/3$ [5] or that the electromagnetic charge of the leptonic singlets are ± 1, 0.

Assuming an unification scheme with the simple group $SU(7)$ and passing through a 331 model with three families, by one step of symmetry breaking, we get the scheme

$$\begin{aligned}
SU(7) \overset{M_U}{\to} \quad & SU(3)_c \otimes SU(3)_L \otimes U(1)_X \\
\overset{M_X}{\to} \quad & SU(3)_c \otimes SU(2)_L \otimes U(1)_Y
\end{aligned} \tag{4.2}$$

the assignment of the electromagnetic charge is of the form

$$\begin{aligned}
Q &= T_{3L} + \alpha Y^a + \beta Y^b + \gamma Y^c \equiv T_{3L} + a\widetilde{Y} \\
a^2 &\equiv \alpha^2 + \beta^2 + \gamma^2
\end{aligned}$$

where Y^a, Y^b, Y^c have the same normalization as the T_{3L}, $\widetilde{Y}$ generators, and they correspond to the $U(1)$ abelian subgroups induced from different subalgebras of $SU(7) \supset SU(n)_L \otimes SU(m)_C \otimes U(1)^a$, where $SU(n)_L \supset SU(2)_L \otimes U(1)^b$ and $SU(m)_C \supset SU(3)_c \otimes U(1)^c$. If the fundamental representation is decomposed as in Eq. (4.1), the most general assignment is

$$Q = diag(q, q, q, b, a, a-1, 1-3q-2a-b). \tag{4.3}$$

And we can identify the charge of $(3,1)$ to be $-1/3$ and those of $(1,2)$ as $(1,0)$, then

$$Q = diag(-1/3, -1/3, -1/3, b, 1, 0, -b) \tag{4.4}$$

where $b = 0, \pm 1$ if we have singlet leptons or $b = \pm 1/3, \pm 2/3, \pm 4/3, \pm 5/3$ if we have singlet quarks.

From the following possible $SU(7)$ maximal subalgebras

$$\begin{array}{rl}
Model\ I & SU(4)_C \otimes SU(3)_L \otimes U(1)^a \\
Model\ II & SU(3)_C \otimes SU(4)_L \otimes U(1)^a \\
Model\ III & SU(6)_L \otimes U(1)^a \to SU(3)_C \otimes SU(3)_L \otimes U(1)^c \otimes U(1)^a
\end{array}$$

we can settle three different assignments for the hypercharge according to the scheme in Eqs. (4.3, 4.4). For example, Model I is known in the literature as the Pati Salam model. The generators are defined by

$$\begin{aligned}
Y^a &= \sqrt{\frac{6}{7}} diag\,(1/4, 1/4, 1/4, 1/4, -1/3, -1/3, -1/3) \\
Y^b &= \sqrt{\frac{1}{3}} diag\,(0,0,0,0,1/2,1/2,-1) \\
Y^c &= \sqrt{\frac{3}{8}} diag\,(1/3,1/3,1/3,-1,0,0,0)
\end{aligned} \tag{4.5}$$

[5] The values $4/3, 5/3$ correspond to exotic quark charges that arise in some 331 models, like model E.

b	Model I			Model II			Model III		
	$\sqrt{\frac{7}{6}}\alpha$	$\sqrt{3}\beta$	$\sqrt{\frac{8}{3}}\gamma$	$\sqrt{\frac{7}{6}}\alpha$	$\sqrt{3}\beta$	$\sqrt{\frac{8}{3}}\gamma$	$\sqrt{\frac{7}{3}}\alpha$	$\sqrt{\frac{1}{12}}\beta$	$\sqrt{\frac{1}{3}}\gamma$
0	-1	$+\frac{1}{3}$	$-\frac{1}{4}$	-1	$+\frac{1}{3}$	$+\frac{1}{4}$	0	$-\frac{1}{3}$	$+\frac{1}{3}$
1	0	1	-1	-1	1	$-\frac{3}{4}$	-1	$-\frac{1}{6}$	1
-1	-2	$-\frac{1}{3}$	$+\frac{1}{2}$	-1	$-\frac{1}{3}$	$+\frac{5}{4}$	1	$-\frac{1}{2}$	$-\frac{1}{3}$
$+\frac{1}{3}$	$-\frac{2}{3}$	$+\frac{5}{9}$	$-\frac{1}{2}$	-1	$+\frac{5}{9}$	$-\frac{1}{12}$	$-\frac{1}{3}$	$-\frac{5}{18}$	$+\frac{5}{9}$
$-\frac{1}{3}$	$-\frac{4}{3}$	$-\frac{1}{9}$	0	-1	$+\frac{1}{9}$	$+\frac{7}{12}$	$+\frac{1}{3}$	$-\frac{7}{18}$	$+\frac{1}{9}$
$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{7}{9}$	$-\frac{3}{4}$	-1	$+\frac{7}{9}$	$-\frac{5}{12}$	$-\frac{2}{3}$	$+\frac{2}{9}$	$+\frac{7}{9}$
$-\frac{2}{3}$	$-\frac{5}{3}$	$-\frac{1}{9}$	$+\frac{1}{4}$	-1	$-\frac{1}{9}$	$+\frac{11}{12}$	$+\frac{2}{3}$	$-\frac{4}{9}$	$-\frac{1}{9}$
$+\frac{4}{3}$	$+\frac{1}{3}$	$+\frac{11}{9}$	$-\frac{3}{2}$	-1	$+\frac{11}{9}$	$-\frac{13}{12}$	$-\frac{4}{3}$	$-\frac{1}{9}$	$+\frac{11}{9}$
$-\frac{4}{3}$	$-\frac{7}{3}$	$-\frac{5}{9}$	$+\frac{3}{4}$	-1	$-\frac{5}{9}$	$+\frac{19}{12}$	$+\frac{4}{3}$	$-\frac{5}{9}$	$-\frac{5}{9}$
$+\frac{5}{3}$	$+\frac{2}{3}$	$+\frac{13}{9}$	$-\frac{3}{2}$	-1	$+\frac{13}{9}$	$-\frac{17}{12}$	$-\frac{5}{3}$	$-\frac{1}{18}$	$+\frac{13}{9}$
$-\frac{5}{3}$	$-\frac{8}{3}$	$-\frac{7}{9}$	1	-1	$-\frac{7}{9}$	$+\frac{23}{12}$	$+\frac{5}{3}$	$-\frac{11}{18}$	$-\frac{7}{9}$

Table 1: Hypercharge definition using the embbeding of $U(1)$ into subalgebras of $SU(7)$ for models I, II, III

In this case Y^b corresponds to the generator T_8 of $SU(3)_L$; and a linear combination of Y^a, Y^c corresponds to $U(1)_X$ of the 331 models. For model II the generators are defined by

$$\begin{aligned} Y^a &= \sqrt{\frac{6}{7}} diag\,(1/3, 1/3, 1/3, -1/4, -1/4, -1/4, -1/4) \\ Y^b &= \sqrt{\frac{1}{3}} diag\,(0, 0, 0, 0, 1/2, 1/2, -1) \\ Y^c &= \sqrt{\frac{3}{8}} diag\,(0, 0, 0, -1, 1/3, 1/3, 1/3) \end{aligned} \tag{4.6}$$

And for model III the generators are defined by

$$\begin{aligned} Y^a &= \sqrt{\frac{3}{7}} diag\,(1/6, 1/6, 1/6, -1, 1/6, 1/6, 1/6) \\ Y^b &= \sqrt{\frac{1}{12}} diag\,(1, 1, 1, 0, -1, -1, -1) \\ Y^c &= \sqrt{\frac{1}{3}} diag\,(0, 0, 0, 0, 1/2, 1/2, -1) \end{aligned} \tag{4.7}$$

By choosing the values for the free parameter $b = 0, \pm 1$ for leptons, and $b = \pm 1/3, \pm 2/3, \pm 4/3, \pm 5/3$ for quarks, the coefficients α, β, γ that determine the hypercharge can be obtained, and they are displayed in table 1. On the other hand, table 2 shows the a^2 normalization factor of the hypercharge according to the b factor.

The next step is to check whether the three family models consider here (models C, D, E, F) can be accomodated properly in a $SU(7)$ GUT. It can be done by comparing the allowed region for a^2 in table 3, with the values of a^2 in table 2. The second column shows the allowed interval of a^2 obtained in previous sections in the absence of a grand unification group, the third column shows the allowed interval by taking into account additional restrictions coming from proton decay ($M_U \gtrsim 2 \times 10^{16}$GeV), that arise when a GUT theory is introduced[6].

The restrictions displayed in table 3 for non SUSY versions of models C, D, F; show that they can be embedded in $SU(7)$ since the bound $a^2 \geq 1/3$ is accomplished by all the values of a^2 in table 2. Model E (the only one with exotic charges studied here) has been ruled out from phenomenological grounds under our scheme. On the other hand, from table 3, we see that the a^2 parameter is strongly restricted in the SUSY versions of these models, and not all of them can be accomodated in $SU(7)$ according to table 2.

b	a^2
0	$\frac{5}{3} \simeq 1.67$
± 1	$\frac{17}{3} \simeq 5.67$
$\pm\frac{1}{3}$	$\frac{19}{9} \simeq 2.11$
$\pm\frac{2}{3}$	$\frac{31}{9} \simeq 3.44$
$\pm\frac{4}{3}$	$\frac{79}{9} \simeq 8.78$
$\pm\frac{5}{3}$	$\frac{115}{9} \simeq 12.78$

Table 2: Hypercharge definition using the embbeding of $U(1)$ into subalgebras of $SU(7)$ for models I, II, III

	Allowed interval for a^2 with no restriction from proton decay	Allowed interval for a^2 with restriction from proton decay
non SUSY C, D, F	$\geq 1/3$	$\geq 1/3$
C (SUSY ZSBS)	$[1.78,\ 3.26]$	$[2.07,\ 3.26]$
C (SUSY XSBS)	$[1.84,\ 1.97]$	$[1.84,\ 1.97]$
D (SUSY ZSBS)	$[2.27,\ 3.26]$	$[2.47,\ 3.26]$
D (SUSY XSBS)	$[1.88,\ 1.97]$	$[1.88,\ 1.97]$
F (SUSY ZSBS)	$[1/3,\ 1.67]$	Excluded
F (SUSY XSBS)	$[1.55,\ 1.82]$	$[1.55,\ 1.82]$

Table 3: Allowed interval for the normalization a^2 for models with three families. In the second column, no restrictions from proton decay are taken into account, while in the third column they are. Models E (SUSY and non SUSY) were discarded from phenomenological grounds.

5 Conclusions

We have studied the possibility of having a 331 model as the only intermediate group between the electroweak scale and the scale M_U in which the three coupling constants unify. We assume that there is no necessarily a simple gauge group at the M_U scale. From the analysis of the renormalization group equations (RGE) we examine different 331 models of one and three families as well as their supersymmetric extensions.

Specifically, we are supposing that the three different couplings unify at certain scale M_U, and a symmetry breaking to a gauge group $SU(3)_L \otimes U(1)_X$ occurs at a lower scale M_X. Then, a second breaking occurs at the M_Z scale to arrive at the SM gauge group. We are also assuming that all particles beyond the SM are getting masses of the order of M_X. Other analyses can be done by supposing that the particle contents are in different thresholds. For our analysis of the RGE, the following conditions are taking into account: ❶ The hierarchy condition given by Eq. (2.19) i.e. $M_X \leq M_U \leq 1 \times 10^{17}$ GeV, should be satisfied. ❷ The condition described by Eq. (2.10) should be

[6] There is another correction coming from the new spectrum introduced, but we have assumed that the new heavy modes do not alter the RGE significantly.

	Allowed interval for M_X		
Model	No SUSY	NSBS	SBS
A	$\left[1.63 \times 10^{16},\ 2.05 \times 10^{16}\right]$	$\left[1.27 \times 10^{8},\ 2.76 \times 10^{13}\right]$	$\left[1.76 \times 10^{14},\ 2.05 \times 10^{16}\right]$
B	$\left[1.63 \times 10^{16},\ 2.05 \times 10^{16}\right]$	No allowed	$\left[1.76 \times 10^{14},\ 2.05 \times 10^{16}\right]$
C	$\left[2.05 \times 10^{16},\ 3.15 \times 10^{16}\right]$	$= 2.76 \times 10^{13}$	$= 2.05 \times 10^{16}$
D	$\left[2.05 \times 10^{16},\ 3.15 \times 10^{16}\right]$	$= 2.76 \times 10^{13}$	$= 2.05 \times 10^{16}$
E	No allowed	No allowed	No allowed
F	$\left[2.05 \times 10^{16},\ 3.15 \times 10^{16}\right]$	$= 2.76 \times 10^{13}$	$= 2.05 \times 10^{16}$

Table 4: Summary of the results obtained in the paper for the six different 331 models studied, as well as their corresponding SUSY extensions

fulfilled, where a is the normalization parameter of the hypercharge Y, that leads to the unification of the couplings at certain scale. ❸ There is no neccesarily a grand unification gauge group at the M_U scale.

In the case of the supersymmetric extensions, the RGE analysis depends on the specific supersymmetric version but also on the supersymmetry breaking scale. In particular, we assume two possible SUSY breaking scenarios: SUSY breaking at electroweak scale and SUSY breaking at the scale of 331 breaking. Although SUSY breaking at electroweak scale is not a realistic scenario, numerical results do not change significantly with respect to the more realistic framework with SUSY breaking at some few TeV's.

Based on the criteria explained above, we can either find an allowed interval for the 331 breaking scale M_X or rule out the model as a possible grand unified theory under the scheme described above. We summarize the results found in this paper in table 4 for the six different 331 models studied here as well as their SUSY extensions (some SUSY extensions that are ruled out have not been included). In many cases, we see that SUSY versions tend to give lower allowed values for the 331 breaking scales than their non SUSY counterparts, it could make SUSY extensions easier to test from the phenomenological point of view. It worths emphasizing that taking into account that the spectrum out of the SM lies at the M_X scale, when assuming that the SUSY breaking occurs at the M_X scale we are in a natural scenario for split Supersymmetry [46]-[47].

On the other hand, we see that models C and D predicts the same allowed intervals for M_X in SUSY and non SUSY versions. However, SUSY versions predict different values for the normalization parameter a^2.

In addition, by finding the allowed values for a^2 we can proceed to see what groups of grand unification could give a value of a^2 lying in the allowed range. Although the introduction of a group at M_U scale introduces new singlets that could lead to tiny changes in the RGE analysis from M_X to M_U, this procedure helps us to figure out what scenarios of grand unification (if any) are possible under our scheme. In particular, we find that some 331 models with three families, can be properly embedded in a grand unification scenario with $SU(7)$, especially in non susy frameworks.

As a matter of perspectives, two loops analysis for RGE can be carried out [48] in order to fit the M_U scale better, especially taking into account the uncertainty in the starting coupling constants at electroweak scale. However, significant changes in the allowed regions obtained here are not expected for a well behaved perturbative regime. On the other hand, it is possible to unify the coupling constant associated to $U(1)$ at string scale instead of the M_U scale [49]-[50].

Finally, it worths saying that the non allowed models are not neccesarily ruled out. We only can say that they cannot produce **UCC** under the scheme in which 331 is the only gauge symmetry between the M_U and SM scales. For instance, it could be possible that they achieve unification by either introducing new physics, or requiring extra breaking steps from the M_U scale to SM scale. By introducing new physics, it is possible to get a lower M_X scale of the order of TeV accesible to the LHC.

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